Lecture 10. **Estimate of quality of regulation in the typical modes and in steady-state mode**

10.1 The short review of the existing estimation methods of quality of regulation

Stability of ACS is necessary, but not sufficient condition for its availability. For example, a system may be stable, but occurs to be insufficiently accurate in its response, to have insufficient speed of reaching the target value or to be unavailable to provide needed smoothness of output coordinate change. Thus, ACS development and research require from us among other things to assure needed quality index of transient process, i.e. speed of operation, oscillation, overshoot and so on, which characterize accuracy and smoothness of the process flow.

In general, if stated simple, operation quality of any ACS is determined by the error value, that equals to the difference between the target and the real values.

For qualitative characteristics determination several criterions were developed; they are subdivided into 4 groups.

The first group contains criteria that use error magnitude measured in different typical modes; they are called fidelity criterion.

The second group combines criteria, which determine stability margin, i.e. how far is the system state from the stability thresholds.

Almost always oscillating threshold is dangerous for a dynamic system. It is caused by the fact that, usually, the urge towards overall gain intensification makes the system approach exactly the oscillating threshold, and then causes self-oscillations.

In the third group reside criteria, which deal with performance speed of a system. Performance speed means the speed of response to control and disturbance actions. The simplest way to estimate such speed is to measure decay time of the transient.

And the last, but far not least, are criteria (*the fourth group*) under the name “integral”. They are used to give some generalized estimation of a system quality and may include factors from all other groups.

10.2 Estimate of quality of regulation in the typical modes

To estimate precision of dynamic system regulation process we use magnitude of errors which appear in different typical modes.

So, let us consider the first group of criteria. We will assume ACS described by a set of linear equations with constant coefficients. The input applied is the unit step function, and the output is a transient process. Consider some typical modes.

1. Static mode (all derivatives are zeroes)

Let the system be given as in fig. 4.1.a. Can we speak about statistic error here?

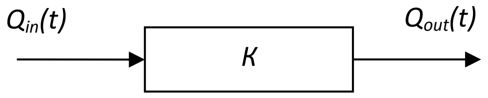

Fig. 4.1a. Open system

All values $Q_{out} = const$, $W(s)\big|_{s=0} = W(0) = K$, where K is a steady-state gain factor (static factor).

Of course, not; there is no source of error in this case. But, if we apply some disturbance *F(t)*, the error is possible (fig. 4.1b).

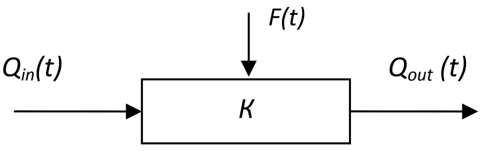

Fig. 4.1b. Open system with disturbances

In this case we deal with steady-state error due to input *F(t)* (disturbance *F(t)).* It is always a rule to indicate the source of error.

Here is another situation (Fig. 4.1c); can we expect any error now?

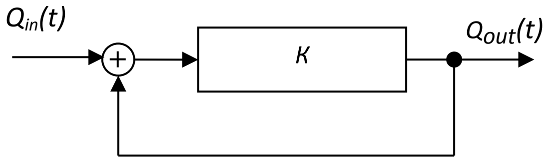

Fig. 4.1c. Closed-loop system

Yes we can, and we should, since the input $Q_{in}(t)$ is combined from usual input signal and negative feedback.

The last case is interesting and important, so *we will consider the influence of negative feedback on steady-state error in more details (if* $Q_{out}(t) = 1(t)$).

Let a system be given, in which all links are static and two external disturbances are presented F_1 *(t)* and F_2 *(t)* (fig.4.2).

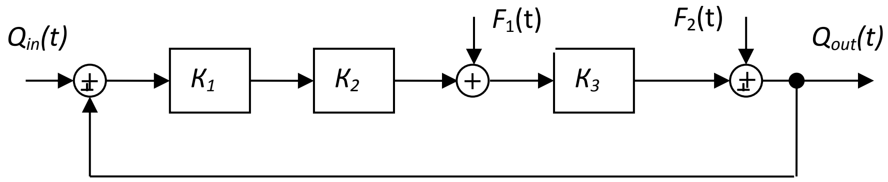

Fig. 4.2. Closed-loop system

Case A. Consider this system without the disturbances applied.

Here

$$W(s) \overset{\Delta}{=} \frac{Q_{out}(s)}{Q_{in}(s)}\ ;\ W_{CL}(s) = \left. \frac{K_1 K_2 K_3}{1 + K_1 K_2 K_3} \right|_{K_0 >> 1} => 1,$$

where $K_0 = K_1 K_2 K_3$ and K_0 is the overall gain factor.

This is an example of steady-state error due to control input (at the finite value K_0).

<u>Statement:</u> *the higher the overall gain factor* $K_0 >> 1$*, the lower the static error value.*

Actually, there is a latent disturbance in all the links: K_i , $K_0 = \prod_{i=1}^{n} K_i$, $0 < K_0 < K_{cr}$ *(here and further* K_{cr} *is critical)*. The proof follows.

The main principle of control is: compare output value with the target one, and apply displacement (the difference between them) with negative sign to the input, in this case: $\varepsilon(t) = Q_{in}(t) - Q_{out}(t)$.

The error transfer function $W_\varepsilon(s)$ is:

$$W_\varepsilon(s) = \frac{\varepsilon(s)}{Q_{in}(s)} = \frac{Q_{in}(s) - Q_{out}(s)}{Q_{in}(s)} = 1 - \frac{Q_{out}(s)}{Q_{in}(s)} = 1 - \frac{K_0}{1 + K_1} = \frac{1}{1 + K_0}$$

at $Q_{in}(s) = 1(t)$.

Steady-state (static) error is

$$\varepsilon = \frac{1}{1 + K_0} \quad . \qquad (4.1)$$

From this follows, that the higher the gain factor K_0, the smaller the static error value ε.

It is a usual practice to raise coefficient K_0 in order to obtain more precise system, but one should always keep in mind the system stability ($K_0 < K_{cr}$).

Case B. The external disturbances are presented.

First of all, rewrite the transfer functions $W(s)$ according to all 3 inputs:

$W(s) = \frac{Q_{out}(s)}{Q_{in}(s)} = \frac{K_0}{1 + K_1}$ (control input),

$W_{F_1}(s) = \frac{Q_{out}(s)}{F_1(s)} = \frac{K_3}{1 + K_0}$ (disturbance F_1),

$W_{F_2}(s) = \frac{Q_{out}(s)}{F_2(s)} = \frac{1}{1 + K_0}$ (disturbance F_2).

Now *the superposition principle* applies:

$$Q_{out}(s) = \frac{K_0}{1 + K_0} Q_{in}(s) + \frac{K_3}{1 + K_0} F_1(s) + \frac{1}{1 + K_0} F_2(s).$$

The latter two summands show contribution of disturbances to error value.

Consider the case, when $K_0 >> 1$. It can be easily seen that output signal is:

$$Q_{out}(s)\Big|_{K_0>>1} = Q_{in}(s) + \frac{K_3}{K_0}F_1(s) + \frac{1}{K_0}F_2(s).$$

Hence, also in this case the steady-state error becomes smaller after increasing the gain factor K_0.

10.3 Estimation of quality of regulation in steady-state mode

Let us consider the control system error at continuous input signal; error the transfer function and expansion error's in series.

Consider the system in fig. 4.3. Here error is $\varepsilon(t) = Q_{in}(t) - Q_{out}(t)$.

The error transfer function is the following:

$$W_\varepsilon(s) = \frac{\varepsilon(s)}{Q_{in}(s)} = \frac{1}{W(s)+1}.$$

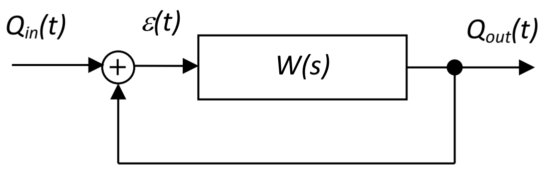

Fig. 4.3. Closed-loop system

The main question is what are the sources of errors in such system?

*First, it is lag***.** To show this, let the transfer function of the open-loop system be the transfer function of the first order aperiodic link K = 1 and $T \neq 0$, e.g. $W(s) = \frac{1}{Ts+1}$. Then mathematical description (in time space) will be written as

$$T\dot{Q}_{out} + Q_{out}(t) = Q_{in}(t)$$

But then $\varepsilon(t) = T\dot{Q}_{out}$, since $\varepsilon(t) = Q_{in}(t) - Q_{out}(t)$.

Conclusion: the error is a function of speed of change of input coordinate, and this is *aftereffect***.**

Secondly, errors originate in characteristics of input disturbances (whether all (n-1) derivatives are not 0). The error transfer function here:

$$W_\varepsilon(s) = \frac{\varepsilon(s)}{Q_{in}(s)} = \frac{1}{1+W(s)} = \frac{1}{1+\frac{N(s)}{M(s)}} = \frac{M(s)}{M(s)+N(s)} = \frac{M(s)}{M_1(s)} = \sum_{i=0}^{\infty} c_i s^i .$$

Rewritten as Maclaurin series:

$$\sum_{i=0}^{\infty} c_i s^i = c_0 + c_1 s + c_2 s^2 + \ldots + c_3 s^n + \ldots \qquad (*)$$

Using (*) and the fact that $\varepsilon(s) = W(s)\mathrm{Q}_{in}(s)$ we will rewrite error function in terms of t instead of s (transformation from image to original):

$$\varepsilon(t) = c_0 \mathrm{Q}_{in}(t) + c_1 \dot{\mathrm{Q}}_{in}(t) + c_2 \ddot{\mathrm{Q}}_{in}(t) + \ldots + c_n \mathrm{Q}^{(n)}{}_{in}(t) + \ldots \qquad (4.2)$$

Accordingly, the system error is determined by the system parameters $c_i\ (i = 0,1,2,\ldots)$ and by the input disturbances parameters $\mathrm{Q}_{in}(t)$. Coefficients in series (4.2) are usually called *error coefficients*: c_o is the static error coefficient; c_1 is the kinetic error coefficient (characterizes speed), c_1 is used when $c_0 = 0$; c_2 is the dynamic error coefficient (characterizes acceleration) etc.

Formula (4.2) is the expansion in series of error in steady-state, where

$$c_i = \left. \frac{\partial^i W(s)}{\partial s^i} \right|_{s=0} \quad \forall i = \overline{1,n}\,;\ c_0 = W_\varepsilon(s)\big|_{s=0}\,.$$

The notion of astaticism (no error)

Astatic Control System

Astaticism is the absence of steady-state (static) error; it is determined by the number of first zero coefficients in series (4.2).

If $c_0 = 0$ indicates nonstatic system of the first order, if $c_0 = 0$ and $c_1 = 0$ is nonstatic system of the second order, and so on.

If at least one of the first coefficients is not equal to zero ($c_0 \neq 0$, $c_1 = 0$), astaticism is impossible, i.e. it is a static system.

The integrating link is the simplest example of astatic link. In particular, electromotor is nonstatic link.

Increasing the number of integrating links leads to several zero error coefficients, and hence, $\varepsilon(t) \to 0$, but the system stability maintenance becomes more complicated.

To conclude, staticism is a measure of relative steady-state (static) error. In some systems stead-state error is undesirable, in those cases the system is designed in the way that makes this error equal to zero, i.e. to nonstatic system.